Spectral Estimation and Color Appearance Prediction of Fluorescent Materials

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ABSTRACT

In this paper, we present a method to estimate the reflected and fluorescent spectral radiance factors of a fluorescent object based on spectrophotometric data without using a monochromator. We use truncated Fourier series to approximate both two spectral radiance factors. Then, based on the measured spectral obtained from a spectroradiometer, the coefficients of the truncated Fourier series are estimated using an weighted least square algorithm. The weighting function is defined as the sum of the CIE standard x, y, and z color matching functions. With the estimated reflected and fluorescent spectral radiance factors, we can predict the color appearance of a fluorescent object under other sources such that the color difference is minimized from viewpoint of human vision.

Keywords: fluorescent objects, fluorescent spectral radiance factor, spectral estimation, color appearance

1. INTRODUCTION

Fluorescent material has been widely applied in various industries. The fluorescent materials have higher lightness and saturation compared with the traditional non-fluorescent materials. The applications of the fluorescent materials include (i) fluorescence dyestuff, such as varnish, toys, printery, and furnishing, (ii) fluorescent ink, such as printing, and warning sign, and (iii) brightening agents such as spin, weave, and papermaking. In industry, the fluorescent materials may be divided into three major categories: brightening agent, the sunlight fluorescent, and inorganic fluorescent.

Though the applications of the fluorescent materials have become more and more important, it is difficult to determine their appearances under different sources. Except for traditional reflected radiance, the fluorescent materials may produce additional radiance called fluorescence, i.e., producing low-frequency radiant energy due to high-frequency energy stimulation. Therefore the chromatic coordinates or the tristimulus values of fluorescent objects will vary dramatically under different light sources. To correctly predict the appearance of a fluorescent object under some specified light source is the main problem for the applications of the fluorescent materials. From the viewpoint of spectrophotometry[1], the fluorescence phenomenon can be characterized by the sum of the one-dimensional reflected spectral radiance factor β_S and the two-dimensional fluorescent spectral radiance factor β_L. The key issue to study a fluorescent material is to determine these two spectral radiance factor functions.

Several methods[1,2,6] had been proposed to determine the spectral reflected radiance factor β_S and spectral luminescent radiance factor β_L. For the one-monochromator method[1], light irradiates on the fluorescent materials and a monochromator is used to control the wavelength range measured by detectors. By using the one-monochromator method, we can obtain the overall spectral radiance factor β_T, which is the sum of the spectral reflected radiance factor β_S and spectral luminescent radiance factor β_L. However, the spectral reflected radiance factor β_S and spectral luminescent radiance factor β_L can not be distinguished from β_T. The experiment of the one-monochromator method is easy to establish. However, as the fluorescent spectral radiance function β_L can not be obtained, this method can be applied to determine the color appearance of the fluorescent object under a specified source.

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For the two-monochromator method[1,2,3,4,5], two monochromators are used as a monochromatic radiation selector and a radiant detector, respectively. When light passes through the first monochromator, the monochromatic radiation at selected wavelength irradiates the fluorescent material. The second monochromator then measure the reflected and re-emitted radiation from the fluorescent material at various wavelengths. Therefore we can obtain a two-dimensional reflected matrix representing the spectrophotometric property of the fluorescent material. The spectral reflected radiance factor β_S is usually determined from the diagonal of the matrix and the spectral luminescent radiance factor β_L from the off-diagonal part. However, significant error for the determined β_S may happen at the wavelengths in the intersection of the excitation band and the emission band of the fluorescent material.

With the two-mode method[2], we can obtain the overall spectral radiance factor and spectral conventional reflectometer value by white irradiating and monochromatic irradiating. Therefore, the overall spectral radiance factor will equal to spectral reflected radiance factor at short-wavelengths. And the conventional reflectometer reading is as the same as the spectral reflected factor at long-wavelengths. Within excitation band and emission band of the fluorescent material, interpolation is used to estimate the spectral reflected radiance factor. Hence, the spectral reflected radiance factor can be determined. And the spectral luminescent radiance factor can be calculated as the difference between the overall spectral radiance factor and the spectral reflected radiance factor. Though, we can determined the spectral reflected radiance, it is difficult to obtain a good estimated spectral reflected radiance factor in both excitation band and emission band of the fluorescent material.

The filter reduction method and the luminescence-weakening method were further proposed[2,6] to provide a better method to estimated the spectral reflected radiance factor in fluorescent excitation and emission region. With the filter reduction method, one can obtain the overall spectral radiance factor by using white light to irradiate the sample. Therefore the overall spectral radiance factor will be equal to the spectral reflected radiance factor at short wavelengths. Then the spectral reflected radiance factor at long wavelengths will be obtained by placing a sharp cut-off filter between the source and the sample. Within excitation band and emission band of the fluorescent material, interpolation is used to estimate the spectral reflected radiance factor. A series of filters are used to reduce the fluorescent emission and to derive an approximation of the spectral fluorescent radiance factor. Because the filter reduction method inducted a series sharp cut-off filter to give an approximation of the spectral reflected radiance factor within excitation band and emission band of the fluorescent material, this method improves estimation accuracy of the spectral fluorescent radiance factor. When samples have a small amount of fluorescence, it is better to use the filter-reduction method. On the contrary, it is preferable to use the luminescence-weakening method[2] in case of large amount of fluorescence. In the luminescence-weakening method, we can obtain the overall spectral radiance factor by using the white light to irradiate the sample. Therefore, the overall spectral radiance factor will equal to spectral reflected radiance factor at short-wavelengths. Then the spectral reflected factor at long-wavelengths will be obtained by placing a complete cut-off filter between the source and the sample. Within excitation band and emission band of fluorescent material, the spectral reflected radiance factor will be deduced by inducting a partly filter just above the minimum of the overall spectral radiance factor. The particular wavelength must choose carefully, otherwise it will induce significant error.

Motta and Farrell[7] used sixteen filtered tungsten lights to generate sixteen sources for which their spectra can be used to approximate most illuminates. Then a matrix is established to present the radiance of the fluorescent material under the sixteen sources. If the vector representing the characteristics based on the sixteen generated sources for a given source can be obtained, then the colorimetric information of the fluorescent material under such a source can be estimated. However, these estimated colorimetric data are not optimized from human vision.

Based on the previous discussions, we now summarize as follows. For the the two-monochromator method and its varieties such as the two-mode method, the filter reduction method, and the luminescence-weakening method , the reflected spectral radiance factor β_S in the middle wavelengths is estimated by interpolating the spectral radiance data of the short and long wavelengths. This rough interpolation cause estimation error of β_S and thus of β_L. Next, the monochromator is used in all the work discussed above. Monochromator is expensive and hard to maintain and operate. The most important thing, which is neglected, is that we should further validate the measured or estimated spectral radiance factors, for example, by predicting the color appearance under some given source. In this paper, we shall construct a method to estimate the reflected and fluorescent spectral radiance factor of a fluorescent object based on spectrophotometric data without using a monochromator. We shall use truncated Fourier series to approximate both two spectral radiance factors. Then, based on the measured spectral distribution, the coefficients of the truncated Fourier series will be estimated using an weighted least square algorithm. The weighting function is constructed by using the CIE standard color matching functions. Then we shall provide an estimated appearance of a fluorescent object under given sources such that the color difference is minimized from viewpoint of human vision.

The remaining of this paper is organized as follows. The spectral properties of fluorescent materials are discussed in Section 2. In Section 3, we shall describe the setup of our experimental environment. A least algorithm, which is used to estimate the spectral reflected radiance factor and the spectral fluorescent radiance factor, and experimental results are described in Section 4. Finally, conclusions are given in Section 5.

2. SPECTROPHOTOMETRIC ANALYSIS AND COLORIMETRIC MEASUREMENT OF FLUORESCENT MATERIALS

Usually, a fluorescent material is made of a substrate by adding some fluorescent agents. Therefore, due to fluorescence, the overall spectral radiance factor $\beta_T(\lambda,u)$ of a material consists of two parts:

$$\beta_T(\lambda,u) = \beta_S(\lambda) + \beta_L(\lambda,u) \tag{1}$$

where $\beta_S(\lambda)$ is the spectral radiance factor reflected by the substrate and $\beta_L(\lambda,u)$ is that accounts for the fluorescence effect of the material. From viewpoint of spectrophotometric analysis made in equation (1), to determine the appearance of a fluorescent material irradiated by different source, it is important to find the two-dimensional spectral radiance function $\beta_L(\lambda,u)$.

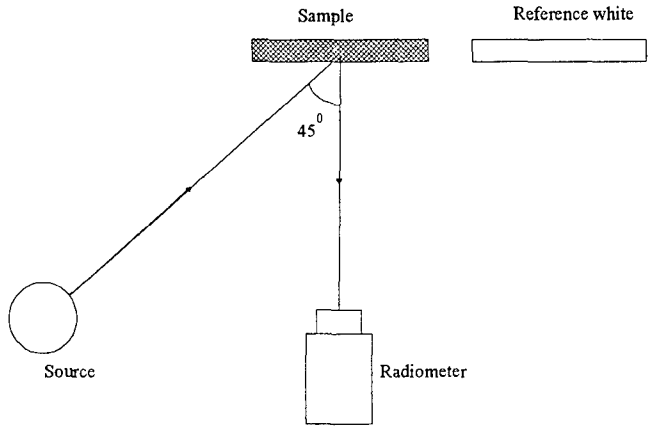

Figure 1: The colorimetric measurement setup

Now we turn to discuss colorimetric measurement of fluorescent materials under the measurement structure shown in Figure 1. Let $S(\lambda)$, $L_{prd}(\lambda)$, $L_{obj\_flu}(\lambda,u)$, and $L_{obj\_sub}(\lambda)$ be the spectral irradiance of the source, the spectral radiance of the reference white, the spectral radiance of the fluorescent agents, and the spectral radiance of the substrate, respectively. Then we define the spectral radiance coefficients of the reference white and the agents of fluorescent sample as

$$q_{prd}(\lambda) = \frac{L_{prd}(\lambda)}{S(\lambda)} \tag{2}$$

$$q_{obj}(\lambda,u) = \frac{L_{obj\_flu}(\lambda,u)}{S(\lambda)} \tag{3}$$

where u is the excited wavelength and λ is the emitted wavelength. Next, the spectral radiance factor of the fluorescent agent is defined as

$$\beta_L(\lambda,u) = \frac{q_{obj\_flu}(\lambda,u)}{q_{prd}(\lambda)} \tag{4}$$

With the definitions made in equations (2) and (3), the spectral function $\beta_L(\lambda,u)$ in (4) can be rewritten as

$$\beta_L(\lambda,u)=\frac{L_{obj\_flu}(\lambda,u)}{L_{prd}(\lambda)} \tag{5}$$

Similarly, the spectral radiance factor of the substrate of fluorescent material is defined as

$$\beta_S(\lambda)=\frac{L_{obj\_sub}(\lambda)}{L_{prd}(\lambda)} \tag{6}$$

Then the overall spectral distribution $C(\lambda)$ measured by the radiometer for the fluorescent sample is given by

$$C(\lambda)=\int_u \beta_L(\lambda,u)S(u)du+\beta_S(\lambda)S(\lambda) \tag{7}$$

By using the spectral function $C(\lambda)$, the colorimetric tristimulus values of the fluorescent under light source can be calculated as

$$X_T=100\frac{\int_\lambda C(\lambda)\bar{x}(\lambda)d\lambda}{\int_\lambda S(\lambda)\bar{y}(\lambda)d\lambda} \quad Y_T=100\frac{\int_\lambda C(\lambda)\bar{y}(\lambda)d\lambda}{\int_\lambda S(\lambda)\bar{y}(\lambda)d\lambda} \quad Z_T=100\frac{\int_\lambda C(\lambda)\bar{z}(\lambda)d\lambda}{\int_\lambda S(\lambda)\bar{y}(\lambda)d\lambda}$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the CIE 1931 standard color matching functions.

3. EXPERIMENT SETUP

3.1 Setup of experimental enviroment

In order to establish colorimetric and radiometric measurements for the fluorescent samples, we set up an experiment as shown in Figure 1. The measurement instrument setup follows the 45/0 geometry. Both the fluorescent sample and the reference white are irradiated by a source and their spectral radiance are measured by a spectroradiometer.

3.2 Selection of Light Sources

To select the sources, we must consider the fluorescence phenomena of the samples. First, the fluorescence is excited within the short-wavelength range of the incident radiant energy. Next, the intensity of the fluorescent spectral radiance factor $\beta_L(\lambda,u)$ is less than that of the reflected spectral radiance factor $\beta_S(\lambda)$. Therefore a source with its power distributed over long-wavelength range can be used to effectively estimate the spectral function $\beta_S(\lambda)$. However, to measure the fluorescent spectral function $\beta_L(\lambda,u)$, we need to select a source with rich excitation over short-wavelength range. We use seven different sources in our experiment. These sources are:

Source 1: a Tungsten lamp,

Source 2: a test lamp from CVI corporation,

Source 3: a Xenon lamp,

Source 4: an approximate of the illuminant D_{65} ,

Source 5: the first test fluorescent lamp ,

Source 6: the second test fluorescent lamp,

Source 7: the third test fluorescent lamp.

The relative spectral power distribution are show in Figure 2, Figure 3, Figure 4, Figure 5, Figure 6 , Figure 7, and Figure 8, respectively. Note that the power spectrums of Source 1 and Source 2 are mainly distributed over long-wavelength range while the excitation of short-wavelength range is richer for Source 3, Source 4, Source 5, Source 6, and Source 7.

3.3 Descriptions of the Fluorescent Samples

We will estimate the color appearance of three kinds of fluorescent samples under different sources. There are three fluorescent samples, which are yellow, pink, and white, respectively, to be tested.

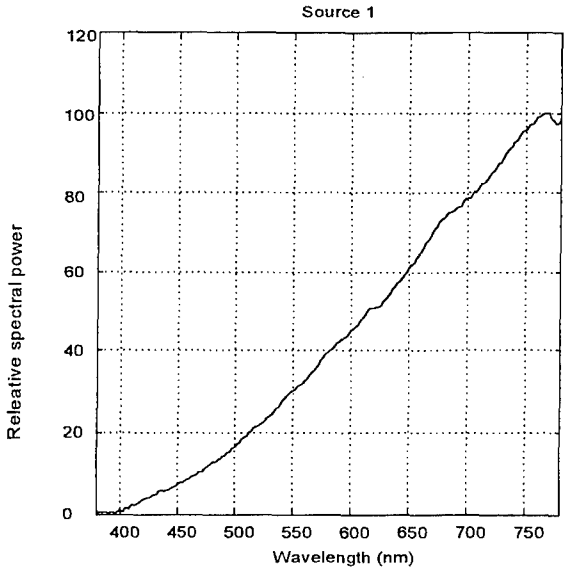

Figure 2: Relative spectral power distribution of Source 1 with 2nm sampling between 380-780nm

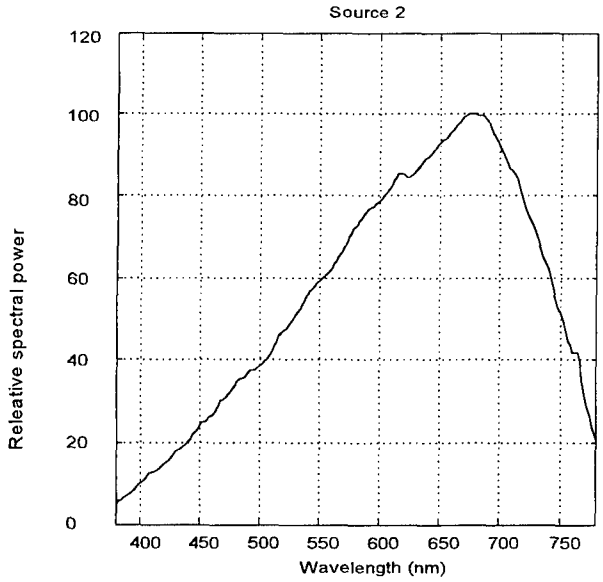

Figure 3: Relative spectral power distribution of Source 2 with 2nm sampling between 380-780nm

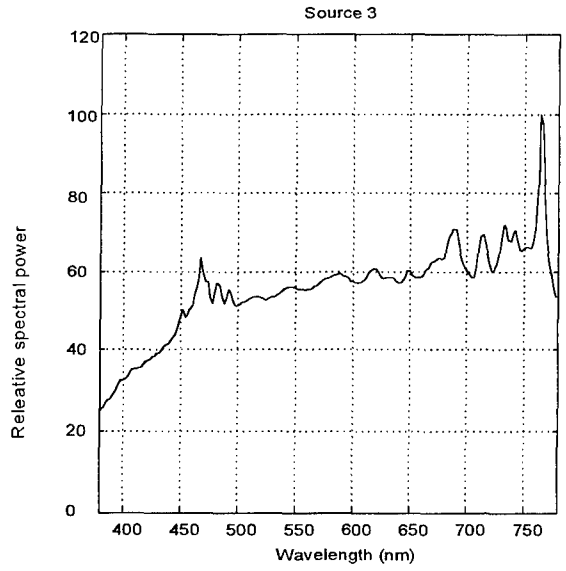

Figure 4: Relative spectral power distribution of Source 3 with 2nm sampling between 380-780nm

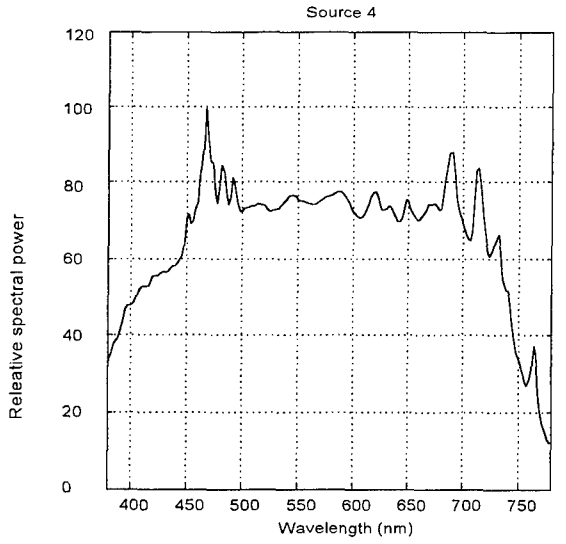

Figure 5: Relative spectral power distribution of Source 4 with 2nm sampling between 380-780nm

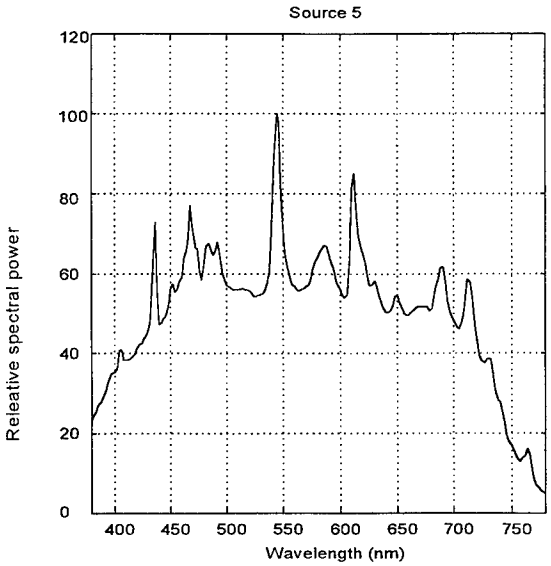

Figure 6: Relative spectral power distribution of Source 5 with 2nm sampling between 380-780nm

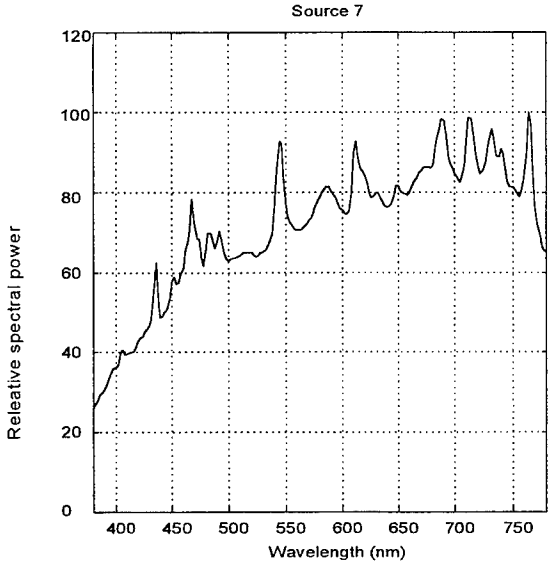

Figure 8: Relative spectral power distribution of Source 7 with 2nm sampling between 380-780nm

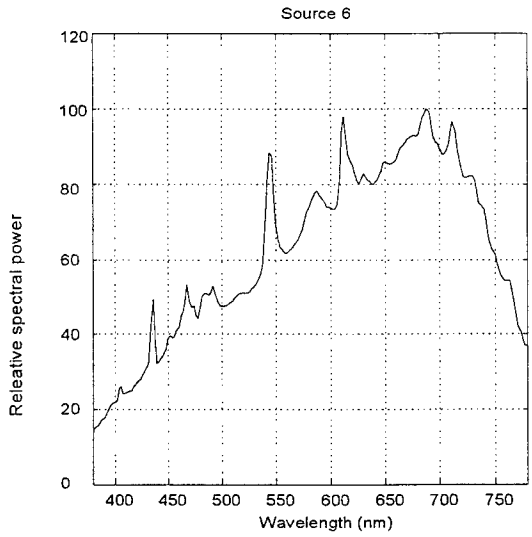

Figure 7: Relative spectral power distribution of Source 6 with 2nm sampling between 380-780nm

3.4 The Observer

The definition of observer in our experiment follows the CIE 1931 Standard Observer. The purpose of using the 45/0 geometry is to avoid re-excitation of the fluorescent materials. We use a spectroradiometer (PR-704) to measure the spectral distribution of the sample. The spectral distribution is sampled every 2nm within the wavelength range from 380nm to 780nm.

3.5 Measurement Procedure

The spectral radiance of the reference white irradiated by the three different sources, Source 1 to Source 3, are measured by the spectroradiometer. Then, the spectral radiance function $C(\lambda)$ for each tested fluorescent sample is measured with respect

to the three light sources. In order to reduce the effect of random fluctuation[1], each spectral distribution is obtained by averaging ten measurements.

4. SPECTRAL ESTIMATION ALGORITHM AND EXPERIMENTAL RESULTS

To estimate the spectral functions $\beta_S(\lambda)$ and $\beta_L(\lambda,u)$, we use Fourier series expansions to represent these two functions. and equation (7) is transformed into the linear regression form in order to apply standard least square algorithm. In particular, to minimize the estimation error of the color appearance of a fluorescent sample, the cost function involved in the least square algorithm is weighted by a function which is the summation of the CIE color-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$. For each fluorescent sample, the measured spectral distribution under Source 1, Source 2, and Source 3, are used in the least squared algorithm to obtain the estimated functions $\hat{\beta}_S(\lambda)$ and $\hat{\beta}_L(\lambda,u)$. These estimated spectral radiance functions are then used to predict the tristimulus of the tested fluorescent samples under Source 4, Source 5, Source 6, and Source 7.

4.1 Least Square Estimation Algorithm

For representing the spectral function $\beta_L(\lambda,u)$, we use a truncated two-dimension Fourier sine series expansion; and for the spectral reflected radiance factor $\beta_S(\lambda)$, a truncated one-dimension Fourier sine series expansion is used as follows. The estimated fluorescent spectral radiance factor is of the form:

$$\hat{\beta}_L(\lambda,u)=\sum_{n=1}^{N-1}\sum_{m=1}^{N-1}\theta_{nm}\sin(\frac{n(\lambda-\lambda_0)\pi}{K})\sin(\frac{m(u-u_0)\pi}{L}) \tag{8}$$

where K=L=400nm, $\lambda_0=u_0=380nm$, and θ_{nm} are the coefficients to be determined. The constant N=26 is the number of sinusoidal function used in the Fourier series expansion with respect to both the u axis and the λ axis. The structure of the estimated reflected spectral radiance function $\hat{\beta}_S(\lambda)$ is given by

$$\hat{\beta}_S(\lambda)=\sum_{\alpha=1}^{\vartheta-1}\delta_\alpha\sin(\frac{\alpha(\lambda-\lambda_0)\pi}{\Phi})+\delta_0 \tag{9}$$

where Φ =400nm and ϑ =51 is the number of coefficients, δ_α and δ_0, in this series to be determined. Note that the number of all the coefficients to be determined in (8) and (9) is 676. This number is much less than that required in the two-monochomator method.

Since the spectral distribution $C(\lambda)$ is measured every 2nm, the estimated total spectral radiance distribution $\hat{C}(\lambda)$ is defined as

$$\hat{C}(\lambda)=\sum_{\substack{u=380\\ u\ even}}^{780}\sum_{n=1}^{N-1}\sum_{m=1}^{N-1}\theta_{nm}\sin(\frac{n(\lambda-\lambda_0)\pi}{K})\sin(\frac{m(u-u_0)\pi}{L})S(u)\Delta u+\left(\sum_{\alpha=1}^{\vartheta-1}\delta_\alpha\sin(\frac{\alpha(\lambda-\lambda_0)\pi}{\Phi})+\delta_0\right)S(\lambda) \tag{10}$$

where $\Delta u=2$ nm. The above expression can be rewritten into linear regression which fits the setup in least square estimation. Define a regression vector $h(\lambda)$ and a parameter vector ϕ as

$$\begin{aligned} h(\lambda) &= \left[f_{1,1}(\lambda)\cdots f_{1,N-1}(\lambda)\ f_{2,1}(\lambda)\cdots f_{N-1,N-1}(\lambda)\ r_1(\lambda)\cdots r_{\vartheta-1}(\lambda)\ r_0(\lambda)\right]^T \\ \phi &= \left[\theta_{1,1}\cdots\theta_{1,N-1}\ \theta_{2,1}\cdots\theta_{N-1,N-1}\ \delta_1\cdots\delta_{\vartheta-1}\ \delta_0\right]^T \end{aligned}$$

where the superscript T denotes matrix transpose, $r_0(\lambda)=S(\lambda)$, and

$$f_{nm}(\lambda)=\sin(\frac{n(\lambda-\lambda_0)\pi}{K})\left(\sum_{\substack{u=380\\ u\ even}}^{780}\sin(\frac{m(u-u_0)\pi}{L})S(u)\Delta u\right),\qquad n,m=1,\ldots,N-1$$

$$r_\alpha(\lambda)=\sin(\frac{\alpha(\lambda-\lambda_0)\pi}{\Phi})S(\lambda),\qquad \alpha=1,\ldots\vartheta-1$$

Then equation (10) can be rewritten into a linear regression form as

$$\hat{C}(\lambda) = h^T(\lambda)\phi \tag{11}$$

For any given estimated vector ϕ, the spectral estimated error at wavelength λ is given by $C(\lambda) - h^T(\lambda)\phi$. Let $\xi(\lambda)$ be a weighting function defined as

$$\xi(\lambda) = \bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda) \tag{12}$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the CIE 1931 standard color-matching functions. The weighting function $\xi(\lambda)$ is used to emphasize the effective wavelength range for human vision. By the definitions of the CIE 1931 standard color-matching functions, we can find that the effective wavelength region for human vision is between 380nm and 750nm which is the union of the supports of the functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$. In order to minimize the color difference from human vision coordinate (X, Y, Z), the weighting function $\xi(\lambda)$ is included in the least square algorithm to specify the wavelength range within which the spectral estimation error is more significant.

There are three sets of spectral distributions irradiated by Source 1 to Source 3 to be used for training the estimated parameter vector $\hat{\phi}$. Each spectral set is sampled every 2nm from 380nm to 780nm. Denote $C_j(\lambda), j = 1,2,3,$ as the measured overall spectral radiance function and $h_j(\lambda)$ as its related regression vector under Source 1, Source 2, and Source 3, respectively. Define a sequence of wavelengths as $\lambda_j = 380 + 2j$ nm, $j = 0,\ldots,200$. Also define the following vectors and matrix as

$$\begin{aligned}
\bar{\xi}^T &= \left[\xi(\lambda_0)\cdots\xi(\lambda_{200}) \quad \xi(\lambda_0)\cdots\xi(\lambda_{200}) \quad \xi(\lambda_0)\cdots\xi(\lambda_{200})\right] \\
\bar{C}^T &= \left[C_1(\lambda_0)\cdots C_1(\lambda_{200}) \quad C_2(\lambda_0)\cdots C_2(\lambda_{200}) \quad C_3(\lambda_0)\cdots C_3(\lambda_{200})\right] \\
\bar{h}^T &= \left[h_1(\lambda_0)\cdots h_1(\lambda_{200}) \quad h_2(\lambda_0)\cdots h_2(\lambda_{200}) \quad h_3(\lambda_0)\cdots h_3(\lambda_{200})\right]
\end{aligned}$$

Now denote $\bar{\xi}_i$ and $\bar{C}_i$ as the i-th entry of the vectors $\bar{\xi}$ and $\bar{C}$, respectively. Let $\bar{h}_i$ be the transpose of the i-th row vector of $\bar{h}$. Then the cost function to be minimized in the least square algorithm is defined as

$$J(\phi) = \frac{1}{2}\sum_{i=1}^{603} \bar{\xi}_i (\bar{C}_i - \bar{h}_i^T \phi)^2 \tag{13}$$

The best parameter vector estimate which minimizes the cost function $J(\phi)$ is denoted as $\hat{\phi}_{LS}$, i.e.,

$$\hat{\phi}_{LS} = \arg\min_{\phi} J(\phi)$$

A recursive algorithm to compute $\hat{\phi}_{LS}$ is given by

$$\begin{aligned}
\hat{\phi}_i &= \hat{\phi}_{i-1} + \frac{P_{i-2}\bar{\xi}_{i-1}\bar{h}_{i-1}}{1+\bar{h}^T{}_{i-1}P_{i-2}\bar{h}_{i-1}}\left[\bar{C}_i - \bar{h}_{i-1}^T\hat{\phi}_{i-1}\right], \quad i = 2,\ldots,603 \\
P_{i-1} &= P_{i-2} - \frac{P_{i-2}\bar{h}_{i-1}\bar{h}_{i-1}^T P_{i-2}}{1+\bar{h}^T{}_{i-1}P_{i-2}\bar{h}_{i-1}}
\end{aligned} \tag{14}$$

where the initial data $\hat{\phi}_1$ can be arbitrarily given and P_0 must be a positive definite matrix. The last estimate $\hat{\phi}_{603}$ is set to $\hat{\phi}_{LS}$.

By the proposed weighted least square algorithm, we can obtain the estimated reflected spectral radiance function $\hat{\beta}_S(\lambda)$ and the estimated fluorescent spectral radiance function $\hat{\beta}_L(\lambda, u)$ for each fluorescent sample. By using equation (10), we can construct the spectral radiance function $\hat{C}(\lambda)$ that estimates the spectral radiance function $C(\lambda)$ for the test fluorescent samples under Source 4, Source 5, Source 6, and Source 7. Comparisons of the estimated overall spectral radiance function

$\hat{C}(\lambda)$ and the measured one $C(\lambda)$ for the three fluorescent samples under different sources are illustrated in Figure 9 to Figure 14. In these figures, the measured spectral function $C(\lambda)$ is well approximated by its estimated one. The approximation error for the measured spectral radiance function $C(\lambda)$ leads to color difference. Color differences in terms of CIELAB Color Difference Formulae for each fluorescent sample under Source 1 to Source 7 are presented in Table 1. The values of ΔE^*_{ab} under Source 1, Source 2, and Source 3 in Table 1 are very small because the measured spectral distribution under these sources are used to train the parameter estimates in the least square algorithm. The estimated spectral functions $\hat{\beta}_S(\lambda)$ and $\hat{\beta}_L(\lambda, u)$ are then used to predict the color appearance of fluorescent samples under Source 4, Source 5, Source 6, and Source7. Since the measured spectral data under Source 4, Source 5, Source 6,and Source7 are not used in the least square algorithm, the values of ΔE^*_{ab} under these sources are usually larger. It is more difficult to decrease the values of ΔE^*_{ab} under Source 5, Source 6, and Source 7 since these sources are mainly based on fluorescent lamps, which have discrete spectral distributions.

ΔE^*_{ab}	Source 1	Source 2	Source 3	Source 4	Source 5-7
Sample 1	0.0010	0.0001	0.0011	3.55	6.73(under Source 5)
Sample 2	0.0013	0.00001	0.0014	3.59	8.81(under Source 6)
Sample 3	0.0130	0.0060	0.0003	4.6	4.41(under Source 7)

Table 1: Color differences of the fluorescent samples under different sources

5. CONCLUSIONS

In this paper, we have presented a method to estimate the reflected and fluorescent spectral radiance factor of a fluorescent object based on spectrophotometric data without using a monochromator. We use truncated Fourier series to approximate both two spectral radiance factors. The total number of parameters to be determined and recorded are much lees than that in the two-monochromator method. With the special weighted least square algorithm, we are able to provide estimated color appearance of test fluorescent objects under different sources such that the color difference is minimized from viewpoint of human vision. With the experiment test made in Section 4, the proposed method can be effectively used to predict color appearance for the applications of fluorescent materials such as fluorescent dyestuff, fluorescent ink, and brightening agents.

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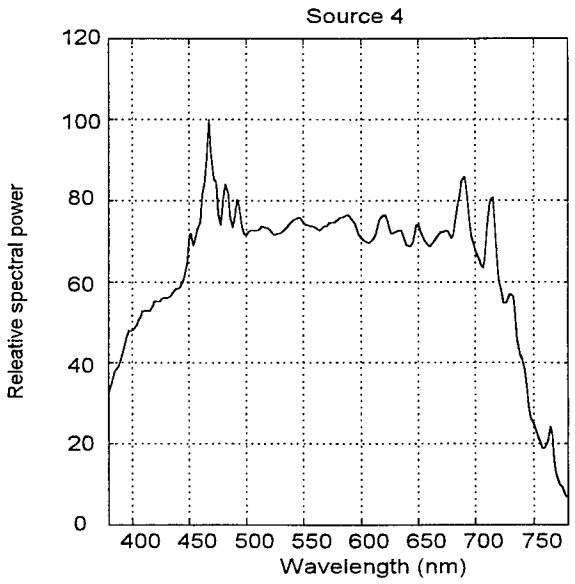


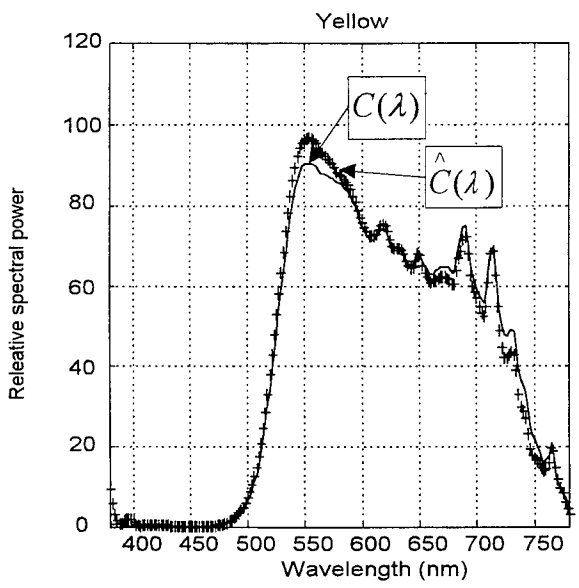
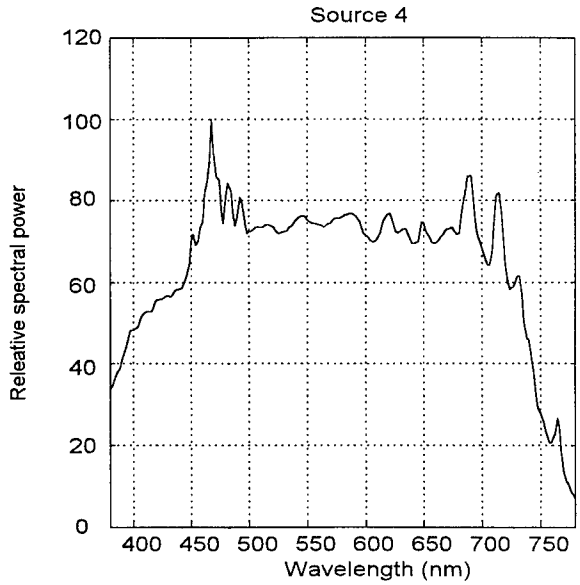


Figure 9: Comparison of $C(\lambda)$ and $\hat{C}(\lambda)$ for the yellow fluorescent sample under Source 4.



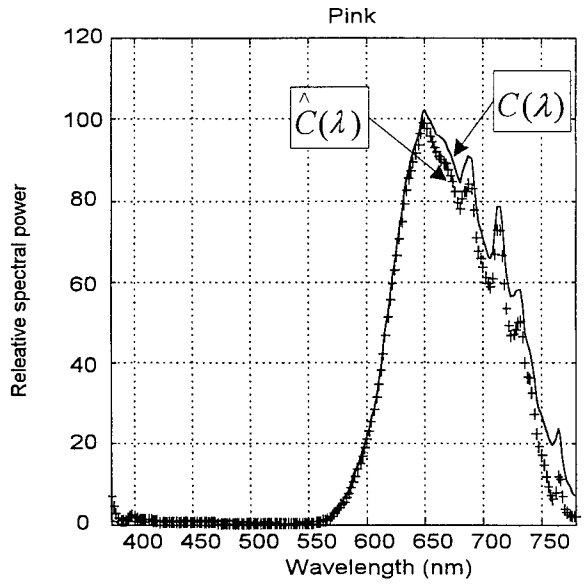


Figure 10: Comparison of $C(\lambda)$ and $\hat{C}(\lambda)$ for the pink fluorescent sample under Source 4.

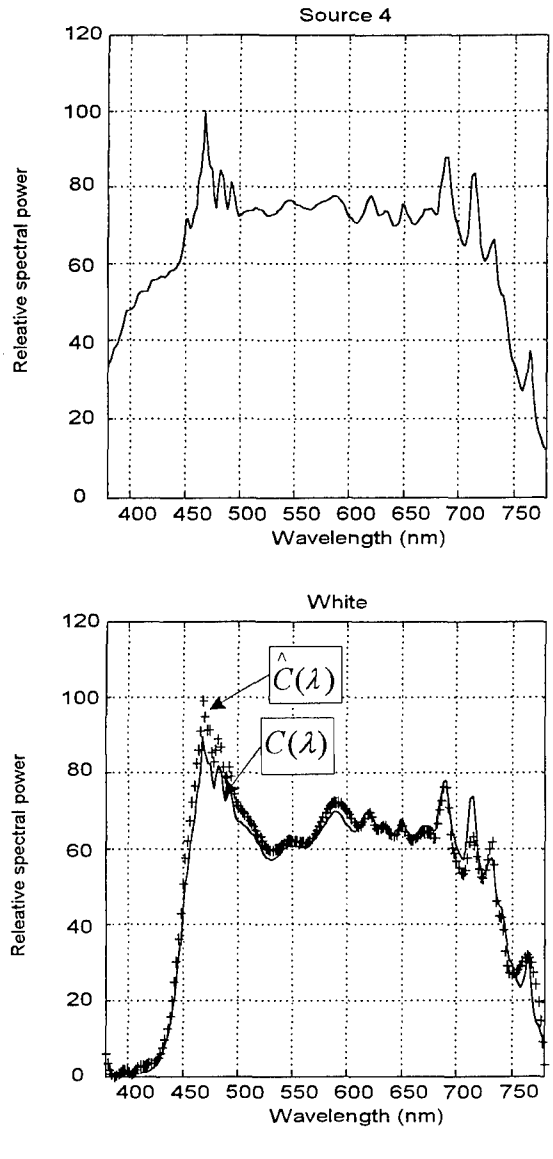


Figure 11: Comparison of $C(\lambda)$ and $\hat{C}(\lambda)$ for the white fluorescent sample under Source 4.

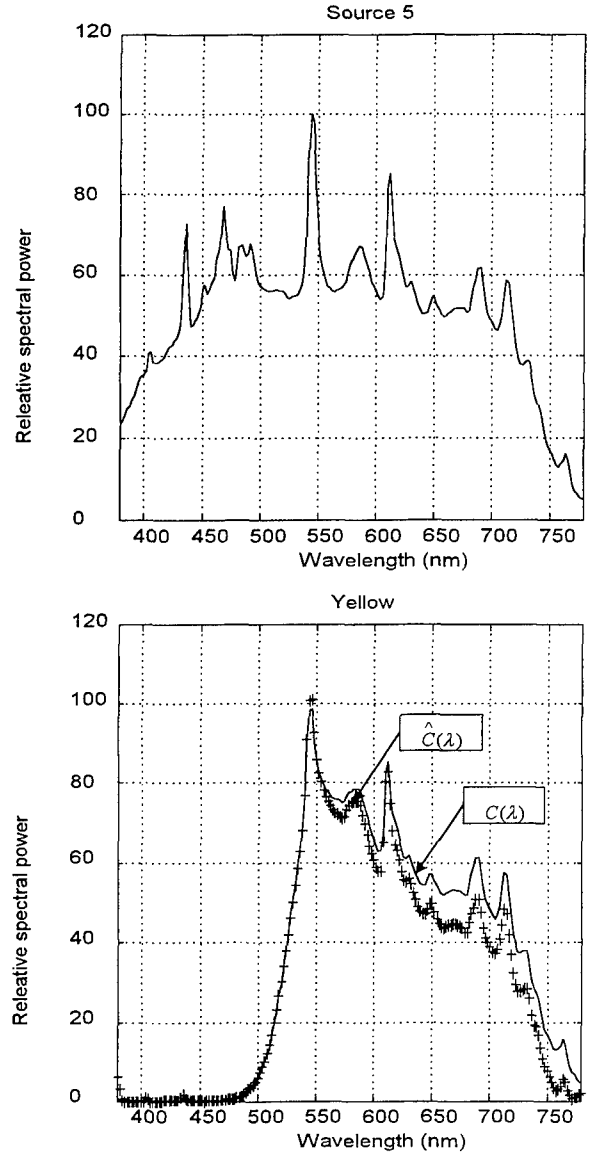


Figure 12: Comparison of $C(\lambda)$ and $\hat{C}(\lambda)$ for the yellow fluorescent sample under Source 5.

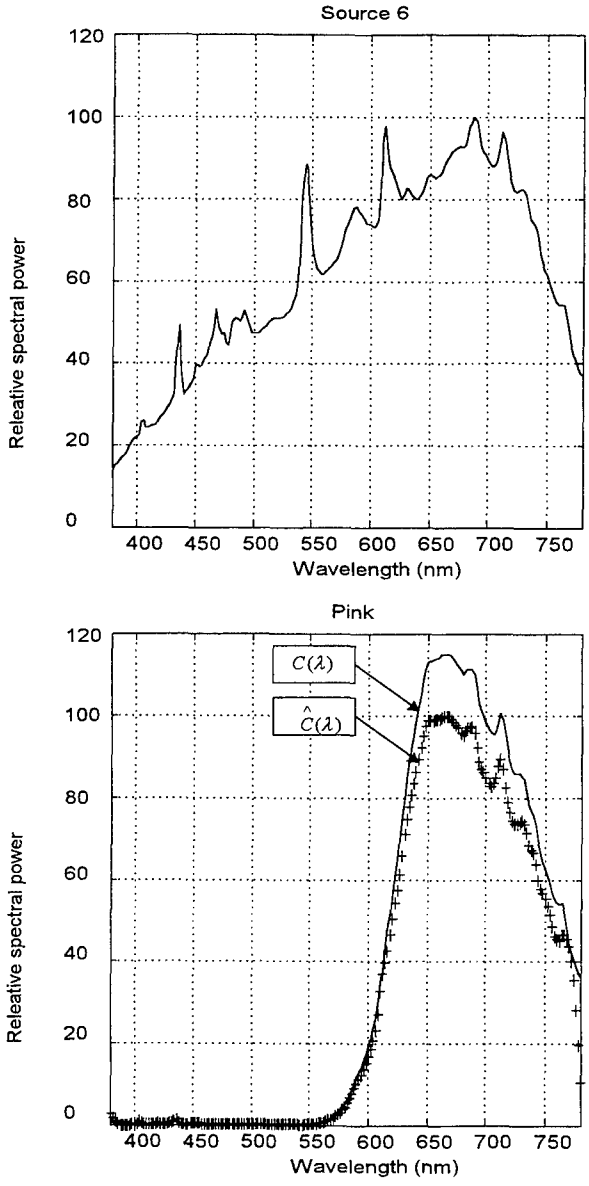


Figure 13: Comparison of $C(\lambda)$ and $\hat{C}(\lambda)$ for the pink fluorescent sample under Source 6.

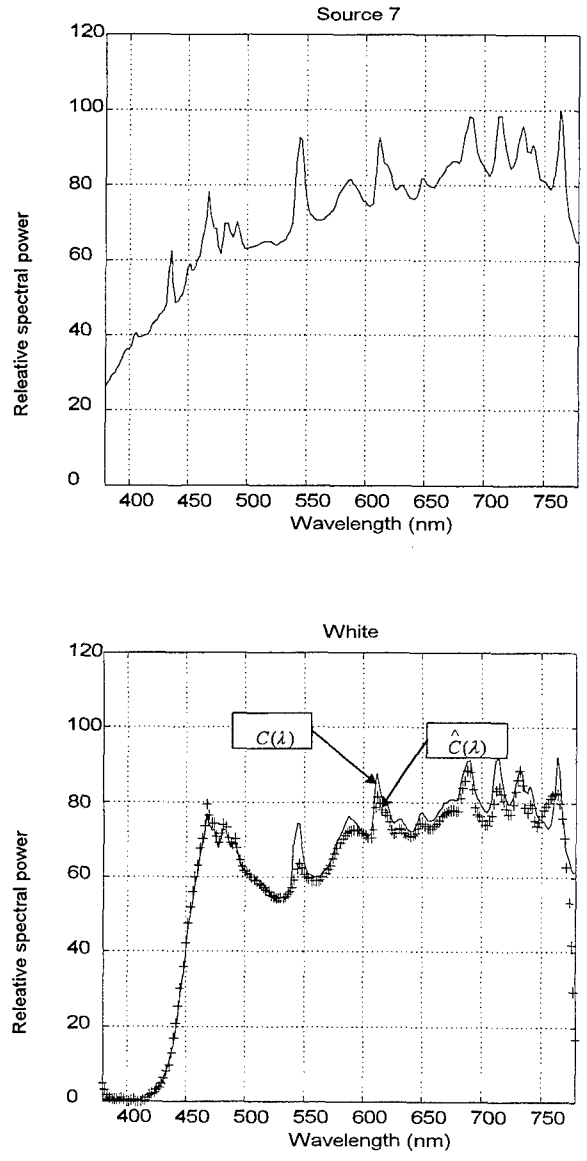


Figure 14: Comparison of $C(\lambda)$ and $\hat{C}(\lambda)$ for the white fluorescent sample under Source 7.